The Central Principle of Deontic Logic

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R. M. Chisholm has noted sometime ago that there is a degree of resemblance between empirical confirmation and ethical requirement; that inductive and deontic logic share certain aspects with one another. He has pointed out that, for instance, just as "p confirms q" does not imply either that p occurs or that q occurs, similarly "p requires q" does not imply that p or q has actually taken place either. Also confirmation is defeasible and may be overridden with additional information, and so is requirement.1

We shall see that Chisholm has merely touched slightly upon the surface of something far reaching that is of utmost significance as well as usefulness. In fact the relation between two logics is very close, so much so that any theorem-like statement in deontic logic can quickly be determined whether or not it is a valid theorem by examining its counterpart in inductive logic to see whether the latter is or is not a valid theorem.

Also it is easy to state the reason why each of these two branches of applied logic should be the replica of the other. The reason is compelling enough not merely to explain retrospectively the various analogies we shall have discovered, but to predict with confidence the concrete manifestations of their close kinship, before having observed them.

I shall attempt to distinguish clearly between two types of cases in which a given theorem-like statement in deontic logic has its inductive counterpart: in the first family of cases the validity of the deontic statement is directly dependent on the validity of the parallel inductive statement simply because the presence or absence of a certain moral obligation is determined by the success or failure of the adequate confirmation of the parallel empirical hypothesis. In the second group of cases the confirmation of the empirical counterpart plays no direct role in deter-

1 American Philosophical Quarterly, 1964.
mining the status of the moral statement, yet the considerations that are
relevant in deciding whether a certain obligation exists may be shown to
bear close resemblance to those that are relevant in the case of
confirmation.

Finally, and perhaps most importantly, great practical benefit is to be
derived from a proper understanding of the kinship between the two dis-
ciplines. Vast amounts of energy and ingenuity have been squandered on
problems that could have been saved. Deontic logic is a novel subject and
its development has been held back because philosophers have run into a
variety of paradoxes. Aziza al-Hibri has in her useful brief survey
described no less than ten paradoxes formulated by different philoso-
phers.² These are symptomatic of the serious troubles the deontic enter-
prise has had from its inception. My main aim is to try to convince the
reader that these great host of problems can be relatively easily dealt with.
The key to their solution lies in the use of the principle always to compare
statements of deontic logic to the identified counterparts and inductive
logic. The latter is a comparatively old and established discipline the ele-
mentary rules of which are fairly well understood.

Thus beginning from Section III we shall start looking at representative
eamples of the various kinds of deontic paradoxes and we are going to
find that in each case the source of the trouble is swiftly identifiable. All we
shall need to do is translate every member of the set of the relevant deontic
proposition into its inductive counterpart. Confronted with the latter the
trouble should as a rule be easily diagnosed owing to our sufficient famili-
arity with elementary confirmation theory. This will lead us at once to the
location of the root of our parallel trouble with the moral statements and to
its rectification.

(II)

In what follows I shall use ‘O(A/B)’ to denote ‘given that B, it ought to be
the case that A’ and to make the various analogies more conspicuous, I
shall use ‘Ac(A/B)’ to denote ‘given that B, the hypothesis A is acceptable’.
As is common practice, we shall regard Ac(A/B) to be the case when
pr(A/B) ≥ n that is, the probability of A given that B is greater than n
which is a number greater than 1/2 and less than 1.

It is well known, for example, that

\( \forall^* \quad Ac(A/B) \rightarrow Ac(A/B \& C) \)

is not a valid theorem of a confirmation theory. The reason is that C may
strongly enough disconfirm A and thus whatever support A may receive


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from B is overridden by the disconfirmation provided by C.

Similarly, the deontic counterpart

\[ (\exists) \quad O(A/B) \rightarrow O(A/B \& C) \]

is not valid either. As mentioned earlier two basically different cases of resemblance between parallel statements in the two logics exist and I propose to use \((\exists)\) and \((\exists^*)\) to bring out as clearly as I can the distinction between them:

**Case 1:** Let \(B = \) At great risk to his own personal safety, K. has saved my life.

I could not show my gratitude to him so far since he has a principle never to accept any gifts from anyone. K. is known however to love surprise birthday parties. He happened to leave his passport with me from which it is evident that his fiftieth birthday is next week.

\(A = \) I invite all K.'s friends to his house where I arrange a lavish surprise birthday party.

I take it that there will be no objection to regard it reasonable that \(O(A/B), \) that is, given that I owe a very large debt of gratitude to K., then now that at least I can offer him something he usually does not refuse, I am morally required to organize a celebration in his honor.

Suppose however that in addition it is also given:

\(C_s = \) K.'s passport is a forgery, several statements made in it are known to me to be false.

Clearly, even though \(O(A/B), \) it is not the case that \(O(A/B \& C_s) \) since there is now good reason to assume that most likely next week it is not K.'s birthday. In this case the failure of \((\exists)\) may be seen as an immediate consequence of the failure of \((\exists^*)\). The reason we regarded \(O(A/B)\) to be true was because B was taken to be supporting a certain empirical hypothesis, but this has now changed. To be more specific:

Let \(F = \) K. is exactly fifty years old next week.

The reason why we regarded A to be morally required was because normally \(Ac(F/B)\) since the vast majority of passports provide reliable information. But now we know that the passport is forged, hence \(~Ac(F/B \& C_s)\) and K.'s birthday is very likely not to occur next week, thus I should cause him embarrassment only, by throwing a birthday party for him. In other words \(O(A)\) would be the case only if the empirical hypothesis F were well established. However, F would be regarded as well established given B alone, but not when we are given B \& C_s. It follows therefore that
O(A) is not the case when we are given both B and C₁, i.e., \( \sim O(A/B & C₁) \) because \( \sim Ac(F/B & C₁) \).

Case 2: Let us have C₂ instead of C₁ where

\[ C₁ = \text{The secret-police is searching for } K, \text{ whom, though a scrupulously upright citizen, they accuse with an assortment of unspecified capital crimes.} \]

Once again it may be said that \( \sim O(A/B & C₂) \). In this case however, the failure of \( (\sim) \) does not arise out of the failure of its precise parallel inductive logic. The hypothesis that next week it is K.’s fiftieth birthday is supported by the existing evidence to the same degree now, as it was before the disclosure of C₁. The various personal data listed in K.’s passport are to be treated to be corresponding to facts, no less now than when we had B only; C₂ has not affected the credibility of any empirical hypothesis B was taken to support. We continue to believe it is K.’s birthday next week, at which time he would greatly enjoy a party honoring him and it is high time I did something to cause him pleasure. However, the introduction of C₂ generates a much stronger obligation to desist, since a widely publicized affair like the one I am planning has a fair chance to come to the attention of the authorities. I am morally obliged to refrain from increasing the probability that K. falls into the hands of his executioners.

It is instructive to note that even though in Case 2 the failure of \( (\sim) \) is not brought about by the failure of its precise inductive counterpart, it still occurs for basically the same reason. \( (\sim) \) is not valid because obligations are defeasible. In Case 1 we saw that the introduction of the C-proposition has supplied strong evidence overriding B’s testimony and thus removing the empirical grounds upon which A would arise. In Case 2 the C-proposition left those grounds intact but has generated a strong counter-obligation, overriding it. Moral obligations just like the credibility of empirical hypotheses may be said to exist not absolutely but relatively to the body of knowledge we have. New information adding to that body, may either affect the empirical grounds in which a given duty was thought to be rooted or introduce a new duty, cancelling it.

(III)

G. H. Von Wright, one of the major architects of deontic logic in a well-known paper, “A New System of Deontic Logic,” enunciates three axioms:

\[
B₁ \quad \sim [O(A/B) & O(\sim A/B)] \\
B₂ \quad O(A & B/C) \leftrightarrow O(A/C & O(B/C))
\]

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B₃ \( O(A/BVC) \leftrightarrow O(A/B) \land O(A/C) \)

To dispel any possible reservations one may have concerning B₃, which may not look as obvious as the first two axioms, Von Wright says:

The following example should convince us of the intuitive plausibility of the third axiom: Suppose we are given the order to see to it that the window is closed should it start raining or thunder. Obviously this is equivalent to being given the order to see to it that the window is closed should it start raining and see to it that the window is closed, should it start to thunder.¹

However in a later part of his paper he shows that with sufficient ingenuity an absurdly paradoxical result may be derived from these seemingly innocuous axioms:

Given that \( p \leftrightarrow (p \land q) \lor (p \land \neg q) \) it follows that

\[
O(A/B) \leftrightarrow O[A/(B \land C) \lor (B \land \neg C)]
\]

But by B₃

\[
O[A/(B \land C) \lor (B \land \neg C)] \leftrightarrow O(A/B \land C) \land O(A/B \land \neg C)
\]

Consequently \( O(A/B) \rightarrow O(A/B \land C) \).

² Hence we have:

(1) \( O(\neg A/C) \rightarrow O(\neg A/B \land C) \) Substitute \( \neg A/A \) and \( B/C \) into (Ⅹ)

(2) \( \neg O(\neg A/B \land C) \rightarrow \neg O(\neg A/C) \) (1) Counterposition

(3) \( \neg [O(A/B \land C) \land O(\neg A/B \land C)] \) Substitute B \( \land C/B \) into B, \( \land C \)

(4) \( O(A/B \land C) \rightarrow \neg O(\neg A/B \land C) \) \( \neg \) Def n of \( \rightarrow \)

(5) \( O(A/B) \rightarrow \neg O(\neg A/B \land C) \) \( \neg \) \( \\& \) (4) Hyp. Syll.

(6) \( O(A/B) \rightarrow \neg O(\neg A/C) \) \( \neg \) \( \land \) (5) \( \\& \) (2) Hyp. Syll.

But of course we cannot entertain the possibility that (6) might be valid since B and C may denote entirely different, logically unrelated circumstances. If (6) were valid it should follow for instance that if it is obligatory to hold a rifle when standing guard in front of Buckingham Palace, then it is permissible to hold a rifle when conducting a religious service in Westminster Cathedral. He proposes to solve the difficulty by pointing out that

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O(A/B) and O(−A/B) are not contradictory statements; logic permits both of them to be true at the same time. Admittedly the two duties cannot simultaneously be carried out, but that means only that we can be in a situation where we have conflicting duties. Such circumstances Von Wright claims are what may be called a moral predicament, like the predicament which arises when a man promises to do the forbidden as Jephta in the Book of Judges. Jephta made a solemn vow (which he was obliged to keep) such that in the end turned out to require the sacrifice of his daughter (which he was obliged to refrain from doing). Thus matters are put right by invalidating \( B_1 \).

Von Wright's position, however, proves to be untenable for at least two reasons. First his rejection of \( B_1 \) appears unreasonable and secondly, which is even worse, the rejection of \( B_i \) is of no help.

Let me begin by elaborating the second objection which is quite decisive on its own. Clearly by invalidating \( B_1 \) Von Wright was only able to prevent the derivation of (6) but not that of (7), which is obtained without the use of any of his other axioms except \( B_1 \). But we have seen in the previous section that it is out of the question for us to accept \( \prec \). It inevitably follows that \( B_1 \) must be invalid.

That this is indeed so is clearly demonstrated by considering the inductive counterpart: \( (B_1^*) \quad Ac(A/B \lor C) \leftrightarrow Ac(A/B) & Ac(A/C) \).

By the inverse formula of probability

\[
p(A/BvC) = \frac{p(BvC/A)}{p(BvC)}
\]

By the Disj. Axiom, assuming \( B \) and \( C \) to be independent

\[
\begin{align*}
&= \frac{[p(B/A) + p(C/A)] \cdot p(A)}{p(BvC)} = \frac{p(B/A \cdot p(A) + p(C/A) \cdot p(A)}{p(BvC)} = \frac{p(A/C) \cdot p(B) + p(A/C) \cdot p(C)}{p(B) + p(C)}
\end{align*}
\]

Let us postulate that \( p(A/B) = n + \epsilon \) and consequently \( Ac(A/B) \) while \( p(A/C) = n - \epsilon \) and therefore \( Ac(A/C) \)

Thus the last expression equals:

\[
\begin{align*}
&= \frac{n \cdot [p(B) + p(C)] + \epsilon[p(B) - p(C)]}{p(B) + p(C)} = n + \epsilon \cdot \frac{p(B) - p(C)}{p(B) + p(C)}
\end{align*}
\]

It turns out therefore that as long as \( p(B) > p(C) \), i.e., as long as \( p(B) - p(C) \) is a positive number, \( p(A/BvC) > n \), which means that

\[
\frac{\epsilon \cdot (p(B) - p(C))}{p(B) + p(C)}
\]
Ac(A/BvC). Thus we have demonstrated the invalidity of (B₁*), since clearly Ac(A/BvC) does not entail Ac(A/B) & Ac(A/C) as in the given example where even though Ac(A/BvC) is true, Ac(A/C) is false.

But given the invalidity of (B₁*) it is impossible for (B₁) to be valid either. For suppose that S in the unique empirical situation such that when S obtains then there is a duty to bring about A. Suppose also that Ac(S/BvC) then it clearly follows that O(A/BvC). But because of the failure of (B₁*) we know that Ac(S/BvC) is compatible with the falsity of Ac(S/B) and Ac(S/C) and therefore also with the falsity of the conjunction of O(A/B) and O(A/C).

(IV)
In Section II we discussed a situation which may be used as one of the endlessly many possible illustrations of the rule to avoid violating (B₁). Having been given Σ (= B & C₃) it might have been thought that we have O(A/Σ), that is, it was obligatory to realize A (= I arrange a birthday party) since such a party would be an appropriate way of showing my gratitude to my benefactors as well as O(¬A/Σ) since Σ also implies that making sure that A is true could endanger the life of a worthy individual. Obviously, however, the conflict is resolved as soon as it arises: the first obligation is trivial in comparison with the second, and I am left with the unambiguous duty not to give away the whereabouts of an innocent man by bringing about A. Our conclusion therefore obeys (B₁): ¬[O(A/Σ) & O(¬A/Σ)].

Thus in all those cases where the potentially conflicting duties do not apply to the same degree the conflict does not last for a single moment: the greater duty overrides the smaller. Axiom B₁ is never violated under such circumstances since it is not actually the case that O(A/B) and O(¬A/B) hold concurrently. It is either that O(A/B) remains only, in case A is the stronger duty, or else O(¬A/B) prevails.

And what about a situation in which the two duties are of comparable magnitude? It seems reasonable to say that in that case the conflicting duties pull me with equal force in opposite directions and therefore cancel one another with the end result that neither an obligation to do A nor to refrain from doing A imposes itself upon me. Once more B₁ is preserved, since in this case the correct thing to say is that ¬O(A/B) and ¬O(¬A/B).

It is worth noting that recently Brian Chellas has expressed his support for Von Wright’s position and offered a short argument in its favor. Chellas has pointed out that the possibility of both A and ¬A being concurrently obligatory.

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is a main feature of some concepts of obligation, that is, often this, for example, that
given moral dilemmas their poignancy.4

There is no disputing the poignancy of moral dilemmas. Many of the
greatest works of literature would not exist without them and there is
hardly anything more riveting than the spectacle of a man being torn by
conflicting obligations as for instance between the demands of the state or
of his religion on the one hand and his commitments to his beloved ones or
personal ideals on the other. But my admission of the centrality of moral
conflicts in the drama of human life does not refute a single word of what I
have just said. It is still true that when of two incompatible moral obliga-
tions one is known to outweigh the other there is no room even for a
moment’s hesitation as it is clear which overrides the other, and when they
are balanced it is unambiguously decided that neither applies. However it
so happens that very frequently it is not clear how to evaluate the magni-
tude of opposing duties and establish their relative weight. These are the
cases where there is a moral predicament generating inner struggle and
arduous deliberation. It, however, in no way affects the validity of (B,).

(V)
I do not wish to cite any further arguments based on the nature of morality
in support of the view that there are no moral dilemmas. The issue has
already received a great deal of discussion and the interested reader may
consult a very recent paper by Earl Conee “Against Moral Dilemmas”5
for arguments of that kind. Let me state, however, that my main reason
for adopting the view I am advocating has been provided by the principle
of the analogy between deontic and inductive logics. According to that
principle anyone who was uncertain about the status of (B,) should con-
sider the status of its counterpart in inductive logic. Now of course few
people would want to deny that

(B,′)  ~[Ac(A/B) & Ac(~A/B)]

is a valid principle. Here too, of course, there are two cases to be distin-
guished. In the first case the amount of evidence for and against A is differ-
ent. Suppose, for example, that ‘A’ stands for Newton’s laws of mechanics
and the time is just before the discovery of Neptune, which accounted for
the apparent discrepancy between the movements of Uranus and the
implications of Newton’s laws. Let ‘B’ stand for what we have just said as
well as the description of all the many phenomena that were so success-
fully accounted for by Newton’s laws. Since by the middle of the nine-

5 Philosophical Review, 1982.
teenth century there has been much evidence to support Newtonian mechanics, it is correct to assert Ac(A/B). However, B has also a component which says that the observed orbit of Uranus has definitely been different from what had been predicted on the basis of A. This amounts to a falsification of A and thus it may well be said that Ac(¬A/B). As we know, however, in the judgment of the scientists of that period it was rational to maintain one’s confidence in A even though that for a while no one could come up with anything adequate to disarm the hostile evidence. They ruled that the empirical support for A was so overwhelming as to dwarf the refuting evidence with the end result that Ac(A/B).

Now a couple of words about situations in which the conflicting degrees of confirmation are of the same magnitude:

Suppose

\begin{align*}
A &= \text{There is life in the solar system beside the earth} \\
B &= \text{There are rings around Saturn}
\end{align*}

Astronomers are fully convinced about the truth of B. If B is true then of course A ⊨ B must be true. By Modus Ponens A & (A ⊨ B) implies B. According to the Hypothetic-Deductive method a hypothesis A is confirmed by a true observation statement B whenever A in conjunction with established auxiliary hypotheses (in this case in conjunction with (A ⊨ B) logically implies B. Consequently Cnf(A/B) should have to be admitted to hold (where ‘Cnf(A/B)’ denotes ‘B raises the credibility of A’) which is rather disturbing. It might be pointed out that there is a difference between Ac(A/B) and merely Cnf(A/B). This however is not much help since if confirmation were conceded acceptability would inevitably follow since if B confirms A so does any number of other established, and entirely irrelevant, empirical statements.

One way of resisting such a conclusion could be to suggest that we abandon the Hypothetic-Deductive method. Remarkably enough there is at least one philosopher who has seriously urged us to do so. Clark Glymour uses this argument in order to show that the view dominant for so long as to what constitutes the essence of scientific method is hopelessly flawed. In fact, however, there is nothing to force upon us any such devastating conclusion. What is correct to say in the context of the present example is that B qualifies as having the tendency to confirm A. But substitute ¬A for A into the two premises and the resulting sentences will be the precise mirror image of the original sentences. For, of course, since B is true, so is also ¬A ⊨ B. Thus ¬A & (¬A ⊨ B) logically implies B and hence for exactly the same reason Cnf(¬A/B) should also be said to be true. Consequently, whatever degree of confirmation B tended to confer

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upon A, it tends to confer precisely the same amount of confirmation upon \( \sim A \). The support provided for A and \( \sim A \) counterbalance and cancel each other. To put it another way: B provides support for (A \& \( \sim A \)) and thus no confirmation, as the probability of an inconsistent statement cannot rise above zero.

A very important point that has emerged from this discussion, one that does not seem to have been noticed before, is that ‘O’ and ‘Ac’ are also duplicates of another in having the common feature of variable strength. Of course, in the case of duties, unlike in the case of probabilities and hence in the case of degrees acceptance, no one has yet devised a numerical scale of measurement and thus there is no quantitative study of moral duties. However, the comparative magnitude of different obligations is of great interest and its study is essential.

A corollary of this shared feature is that just as in the context of confirmation when the support for A clearly outstrips the support of \( \sim A \) then the latter is disregarded so in the moral context when the reasons for doing A distinctly outweigh the reasons for not doing it the latter are ignored. We are then left with unequivocal obligation to bring about A. On those occasions when we find equal and opposing tendencies, in both cases we apply the principle of the cancellation of symmetrical opposites and the situation is as if none of the opposing considerations existed.

Thus we have had major testimony as to how truly significant as well as useful is the resemblance between deontic and inductive logic. Some philosophers have drawn attention to certain similarities between deontic and modal logic. But the two exemplifications of kinship are entirely different. Not only is there no compelling explanation why modal logic should be analogous to the logic of obligations, and not only has no one so far suggested for what practical purpose the analogy might be exploited but also upon a closer look the dissimilarities between deontic and modal logic appear to outweigh the similarities between deontic and modal logic. While for example there is no system of modal logic in which \( \Box A \) does not entail A, OA is of course compatible with A. It is to be noted on the other hand that here again inductive logic mirrors the logic of obligation. A proposition A may be highly probable relative to what we happen to know, in which case Ac(A) is true, nevertheless A may turn out to be false. Now we have encountered another basic feature which the logic of obligations and the logic of confirmation share with each other, a feature absent in modal logic. A proposition is either necessary or not; necessity, unlike credibility and moral duty, does not come in indefinitely many degrees.
The reader may well be wondering how this might be possible: surely to assert that the duty to do A as well as the duty to do B, is no more and no less than the duty to do A and B, is merely to assert a truism! Furthermore, is anyone known to have denied or even merely queried the validity of $\langle B_2 \rangle$?

The answer to the last question is that I know of no one who has voiced any objection to $\langle B_2 \rangle$ and in fact I know of quite a number of philosophers who have explicitly declared the validity of $\langle B_2 \rangle$. I have also found that other experts in the field like von Kutschera, Hans Lents, David Lewis, Bengt Hansson, and Aziza a-Hibri have approved of it.

The answer to the first question is that admittedly $\langle B_2 \rangle$ looks as if it were trivially true yet a brief glimpse at the situation in inductive logic is bound to convince us otherwise. Consider

$$\langle B_2^* \rangle \; \text{Ac}(A/C) \; \& \; [\text{Ac}(B/C) \rightarrow \text{Ac}(A \; \& \; B/C)]$$

As is known the joint probability of A and B is less than that of A or B alone, in all those cases where each is less than 1 and neither entails the other. In consequence of this, the probability of A & B may be less than n even though the probability of A as well as that of B is equal or more than n. In that case however we would have

$$\text{Ac}(A/C) \; \& \; \text{Ac}(B/C) \; \& \; \neg \text{Ac}(A \; \& \; B)$$

This feature of inductive logic has been aptly described by S. R. Levy recently by saying that

\ldots rational belief in each of a series of propositions does not necessarily carry over into a
rational belief of the conjunction of those proportions . . . 

It should be clear by now that the invalidity of $(B_1)^*$ is bound to render $(B_2)$ invalid too. For suppose 'a' and 'b' denote propositions describing empirical situations and assume that $a \rightarrow OA$ and $b \rightarrow OB$. Furthermore let $p(A/C) \geq n$ as well as $p(b/C) \geq n$ while $p(a \& b/C) < n$. In this case then it is rational to believe in each one of $a$ and $b$ but not in the conjunction of those propositions. In our terminology

$$\text{Ac}(A/C) \& \text{Ac}(b/C) \& \sim \text{Ac}(a \& b/C)$$

Now given that $a \rightarrow OA$ and $b \rightarrow OB$ it may be said to follow that $\text{Ac}(A/C) \rightarrow O(A/C)$ and $\text{Ac}(b/C) \rightarrow O(B/C)$. It is also true of course that $\text{Ac}(a \& b/C) \rightarrow O(A \& B/C)$, but as we have seen we do not have $\text{Ac}(A \& b/C)$. Thus we do not have $O(A \& B/c)$ either!

(VII)

If the ideas advanced so far are valid then it is to be expected that we should be able to develop, with their aid, a complete system of deontic logic. It is indeed fairly obvious now how we would proceed toward such an end. We would begin by compiling a list of theorems concerning the acceptability of empirical hypotheses based on elementary probability theory. Then we simply translate each item on that list into its deontic counterpart. The following list contains some theorems of induction logic:

(1*) $\text{Ac}(A \& B/C) \rightarrow [\text{Ac}(A/C) \& \text{Ac}(B/C)]$

(II*) $[\text{Ac}(A v B/C) \& \text{Ac}(\sim A/C)] \rightarrow \text{Ac}(B/C)$

(III*) $\sim[\text{Ac}(A/C) \& \text{Ac}(\sim A/C)]$

(IV*) $[\text{Ac}(A/BvC) \& \text{Ac}(A/\sim BvC)] \rightarrow \text{Ac}(A/C)$

(V*) $[\text{Ac}(A/B) \& \text{Ac}(A/C)] \rightarrow \text{Ac}(A/BvC)$

(VI*) $\text{Ac}(A/B \& C) \rightarrow \text{Ac}(A v B/C)$

(VII*) $\text{Ac}(A/BvC) \rightarrow [\sim B \rightarrow \text{Ac}(A/C)]$

(VIII*) $\text{Ac}(A&B/C) \rightarrow \text{Ac}(A/B \& C)$

By applying the fundamental principle of the analogy between deontic and inductive logics we at once obtain from the above:

(I) $O(A \& B/C) \rightarrow [O(A/C) \& O(B/C)]$

Let us have a closer look at (VIII) which may seem less obviously true than perhaps any of the earlier theorems. A number of philosophers subscribe to (VIII) among them Bengt Hansson and Azizah al-Hibri. The latter explains that (VIII) is highly intuitive since it allows for the possibility that a complex obligation be satisfied in stages, without altering that complex obligation at any stage. We of course have arrived at (VIII) simply because we found it to be a counterpart of (VIII*). Should we be called upon to do so we could offer a conclusive justification of the latter:

By the Conjunctive Axiom of Probability

$$\text{pr}(A \& B/C) = \text{pr}(B/C) \cdot \text{pr}(A/B \& C)$$

and of course $\text{pr}(B/C) \leq 1$

hence $\text{pr}(A \& B/C) \leq \text{pr}(A/B \& C)$

Thus given that $\text{Ac}(A \& B/C)$ which means that $\text{pr}(A \& B/C) \geq n$ it inevitably follows that $\text{pr}(A/B \& C) \geq n$ and hence $\text{Ac}(A/B \& C)$.

In earlier sections we have used this same method to determine that certain expressions that might plausibly be taken to represent theorems of deontic logic do not represent valid theorems. Let me cite here two more examples of such expressions. Bengt Hansson in his important survey article advances

$$(\Phi) \quad O(A/C) \& \sim O(\sim B/C) \rightarrow O(A/B \& C)$$

as an acceptable theorem on which he comments by saying:

An obligation remains an obligation if one does something permitted.

Now (\Phi) and Hansson’s brief defense might look plausible to some, but anyone aware of the central role of our methodological principle will not decide the matter before examining the inductive counterpart:

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\[(\Phi^+) \quad \text{Ac}(A/C) \& \neg \text{Ac}(\neg B/C) \rightarrow \text{Ac}(A/B \& C)\]

This expression however would be rejected by all inductive logicians. If for example ‘C’ stood for the law of gravity and the law ascribing fragility to glass objects and ‘A’ denoted ‘the glass bottle I throw out of the window of my fifth floor apartment is going to break’, then it is rational to contend that \text{Ac}(A/C). Let us also suppose that

\[B = \text{The pavement beneath my window is covered with thick cushions.}\]

Clearly \(\neg \text{Ac}(\neg B/C)\) since no law of physics is relevant at all to the question whether or not there are cushions beneath my window. Given B, then the discarded bottle is going to land on a soft surface and is unlikely to shatter. Hence \text{Ac}(A/B \& C)\ is false. The invalidity of \(\langle \Phi^* \rangle\) must of course alert us to the fact that \(\langle \Phi \rangle\) is not valid either. After some search for a suitable example illustrating this, the invalidity of \(\langle \Phi \rangle\) becomes indeed fully evident:

Let it for instance be given that

\[C = \text{Jones was robbed a little while ago}\]
\[A = \text{Fred, Jones’ neighbor goes to the assistance of the latter}\]

It is reasonable to assert that \text{O}(A/C). Let us now suppose that

\[B = \text{Fred just had a major operation two days ago and finds it very painful even just to sit up in his bed.}\]

Quite obviously \(\neg \text{O}(\neg B/C)\) since it is by no means forbidden for Fred to have a major operation in the beginning of the week just because his neighbor is robbed in the middle of the week. In fact B may be said not only permissible but even obligatory if it were given that the major operation was the only way to save Fred’s life. But no person in Fred’s condition can be required to exert himself in order to go to the assistance of anyone. Hence \text{O}(A/B \& C)\ is false.

I conclude with an example that we shall require in our discussion in the last section of this paper. Consider:

\[(\Psi) \quad \text{O}(A/B) \& \text{O}(B/C) \rightarrow \text{O}(A/C)\]

It might well seem that moral obligations are transitive and thus \(\langle \Psi \rangle\) is valid. It appears quite plausible that if C generates the duty to bring about B while B itself generates the duty to bring about A then C is bound to give rise, via B, to the obligation to see to it that A is true. However, as is well known, the inductive counterpart of \(\langle \Psi \rangle\),

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\((\Psi^*)\) \(Ac(A/B) \& Ac(B/C) \rightarrow Ac(A/C)\)

is not a valid expression, since the probability of \(A\) given \(B\) with certainty may be large enough but not when \(B\) is less than certain. Thus we must conclude that \((\Psi)\) is not valid either.

Once we become aware of the invalidity of \((\Psi)\) it is comparatively easy to find examples showing conclusively that this is so.

Let \(B =\) Fred’s surgeon performs a major operation on him

\[C =\) Fred is suffering from acute \(x.\]

A knowledge of the nature of \(x\) may make it obvious that \(O(B/C)\). Let us also suppose that

\[A =\) Fred’s surgeon makes sure that Fred is administered heavy anesthetics.

Assuming that it is very cruel to operate on a person alive to pain, it is correct to maintain \(O(A/B)\). However \(O(A/C)\) cannot be asserted since it is highly undesirable to administer heavy anesthetics to anyone unless he was certain to undergo an operation. It is of course possible for an endless variety of reasons that in spite of Fred’s urgent need, his surgeon is not going to perform the required operation. The surgeon may have broken his arm or was detained by the authorities as a suspected drug dealer or because Fred is at the moment having very high temperature. For whatever reason \(B\) turns out not to be the case, clearly not only does it follow that the consequent of \((\Psi)\) is false but that positively it is obligatory to make sure that \(\sim A\).

\((VIII)\)

Let us return now to the subject of the paradoxes that have arisen in deontic logic. We have seen that in the case of Von Wright the source of the paradox was his mistaken adoption of an untenable axiom. There are however paradoxes where the root of the trouble is to be sought elsewhere. We shall now look at some such paradoxes that have widely been discussed in the last few years. The so-called Good Samaritan paradox belongs to one group of paradoxes of this kind. One of its versions runs as follows:

\(a:\) If the Good Samaritan helps Jones who has been robbed then

Jones has been robbed

\(b:\) It is forbidden that Jones be robbed
Let \( A = \) The Good Samaritan helps Jones
\( B = \) Jones was robbed

It has been suggested that \( a \) and \( b \) is to be symbolized:
\[
\begin{align*}
a' &: (A \& B) \rightarrow B \\
b' &: O(\sim B)
\end{align*}
\]

and consequently, given the rule of inference

\[
\text{If } p \rightarrow q \text{ then } O(\sim q) \rightarrow O(\sim p)
\]

\( a' \) together with \( b' \) yield \( O[\sim (A \& B)] \) which amounts to the absurd conclusion that it is forbidden for the Good Samaritan to help Jones who was robbed.

According to the key methodological rule advocated in this paper the effective way to discover where precisely lies the error that has generated the paradox is to translate the relevant propositions into their inductive counterpart.

Now of course the counterpart of \( b \) is:
\[
\beta: \text{The hypothesis } \sim B \text{ is well established}
\]
in symbols: \( \beta': Ac(\sim B) \)

Combining \( a' \) and \( \beta' \) yields \( Ac[\sim (A \& B)] \), which means ‘The hypothesis that it is false that the Good Samaritan helps Jones who has been robbed, is well established’. Now this result will also appear unacceptable as for example if we are given the truth of the universal generalization that the Good Samaritan helps every person who has been robbed.

In this case, however, we shall inevitably be led straight to be the only possible source of our problems. The corresponding rule of inference we have employed in the inductive case was:

\[
\text{If } p \rightarrow q \text{ then } Ac(\sim q) \rightarrow Ac(\sim p)
\]
is of course absolutely above suspicion. It follows directly from the basic axioms of probability that if \( p \rightarrow q \) then \( pr(q) \geq pr(p) \) and thus \( pr(\sim q) < pr(\sim p) \). Now ‘\( Ac(\sim q) \)’ has been defined as ‘\( pr(\sim q) \geq n \)’ and if \( pr(\sim q) \geq n \), then \( pr(\sim p) > n \) that is \( Ac(\sim q) \rightarrow Ac(\sim p) \). In addition to this we are to be reminded that no other rule of inference or theorem has been employed, so nothing can be wrong with the derivation. It can thus not be denied that \( Ac[\sim (A \& B)] \) is a correct conclusion.

Unavoidably therefore we are made to realize that an appearance of paradox must have been generated because we did not look at the conclusion in a correct way. Given the universal generalization mentioned
before, that is, given that \( \text{pr}(A/B) \) is virtually one and the same time given also \( \text{Ac}[\neg(A \& B)] \) we are forced to conclude that the reason why it is unlikely to be true that the Good Samaritan does \textit{not} help Jones who has been robbed because Jones happens not to have been robbed. After all the statement \( \text{pr}(A/B) = 1 \) is compatible with \( \neg(A \& B) \) since that conjunction may be false not only when \( A \) is false while \( B \) is true but also when \( B \) is false.

In the deontic context we shall not be prevented from subscribing to the very reasonable proposition \( \text{O}(A/B) \) — which of course is the counterpart of \( \text{Ac}(A/B) \) we were given to be true — that is we shall believe as is proper that it \textit{is} morally obligatory that the Good Samaritan should help Jones \textit{given} that Jones was robbed. At the same time it is quite true that \( \text{O}[\neg(A \& B)] \) for it is indeed obligatory to ensure that \( (A \& B) \) is false. In general of course a conjunction can be made to turn out to be false in a variety of ways by making both or only the first or only the second conjunct false. Given that \( \text{O}(A) \), it follows that the falsity of \( (A \& B) \) must be brought about by making sure that \( B \) is false.

(IX)

Finally I propose to have us look at a paradox raised by Chisholm in 1963 and which has been referred to as the most “famous and perhaps the most worrisome paradox to Standard Deontic Logic (SDL)” as recently as in 1981 by J. W. Decew\(^{10}\) who is sceptical whether so far any adequate solution to it has been provided.

Chisholm has pointed out the following four English sentences are intuitively consistent.

(1) It ought to be that a certain man go to the assistance of his neighbors.

(2) It ought to be that if he does go then he told them he is coming.

(3) If he does not go then he ought not to tell them that he is coming.

(4) He does not go.

These sentences, however, are symbolized in SDL in the following manner:

\[
\begin{align*}
(1a) & \quad \text{Og} \\
(2a) & \quad \text{O}(g \supset t) \\
(3a) & \quad \neg g \supset O \sim t \\
(4a) & \quad \neg g
\end{align*}
\]


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Now (1a) and (2a) yield by the deontic distribution axiom, Ot, while (3a) and (4a) yield by Modus Ponens O ⊃ t. Thus it follows that both t and ¬t are obligatory. This incidentally should be quite unacceptable even to philosophers like Von Wright and Chellas who do not admit that for every p, ¬(Op & O¬p) is true. After all here we are confronted by a straightforward simple situation which is devoid of elements that could give rise to moral conflict or predicament, in any sense of that term.

Thus once more we have been presented with an important opportunity for employing the central methodological principle of deontic logic. We begin by translating each one of propositions (1)-(4) into its inductive counterpart. If the fundamental thesis of this paper is correct then we should expect to be led to a parallel inductive paradox. As before, because of our much greater mastery of inductive logic, we are likely to notice immediately in the latter case which one of our steps was erroneous and thus responsible for generating the difficulty. This of course will enable us to resolve both paradoxes. After having said this we shall perhaps not find it too surprising to see that indeed there is a full-fledged inductive replica to Chisholm’s paradox. In order to demonstrate it we formulate the following four sentences:

(1*) The hypothesis that a certain man goes to the assistance of his neighbors, is acceptable.
(2*) The hypothesis that if he does go then he tells them he is coming, is also acceptable.
(3*) If it is true that he does not go then the hypothesis that he does not tell them he is coming, is acceptable.
(4*) He does not go.

These sentences, just as Chisholm’s, are consistent. It should be clear for instance, that (1’) and (4’) are not incompatible since as pointed out before, what is highly probable may yet be false. Now we symbolize the four sentences:

(1a*) Ac(g) [assuming the probability of g to be more than high enough, e.g., pr(g) = m where m = 3n]

(2a*) Also pr(g ⊃ t) = m
(3a*) ¬g ⊃ Ac(¬t)
(4a*) ¬g
But (1a*)-(4a*) may be shown to be inconsistent since on the one hand we have:

(i) \( \text{pr}(g \supset t) = \text{pr}[^{\sim}(g \& \sim t)] \) \quad \text{Defn. of '}'

(ii) \( = 1 - \text{pr}(g \& \sim t) \) \quad \text{Disj. Axiom of Prob.}

(iii) \( = 1 - \text{pr}(g) \cdot \text{p}(\sim t/g) \) \quad \text{Conj. ' ' ' ' '}

(iv) \( = 1 - \text{m} \cdot \text{p}(\sim t/g) = \text{m} \quad (i) \& (1a*) \& (2a*) \)

(v) \( \text{pr}(\sim t/g) = \frac{1 - \text{m}}{\text{m}} \quad (iv) \text{Algebra} \)

(vi) \( \text{pr}(t/g) = \frac{2\text{m} - 1}{\text{m}} \quad (v) \& \text{Disj. Ax.} \)

(vii) \( \text{pr}(t) = \text{pr}(t \& g) + \text{p}(t \& \sim g) \) \quad \text{Theorem of Probability}

(viii) \( \text{pr}(t) - \text{p}(g) \cdot \text{pr}(t/g) + \ldots \ldots \) \quad (vii) \text{Conj. Axiom}

(ix) \( = \text{m} \cdot \frac{2\text{m} - 1}{\text{m}} + \ldots \ldots \)

\( = 2\text{m} - 1 + \ldots \ldots \geq 3\text{n} - 1 \)

Since \( n > 1/2, 3n > 1 + n \)

and thus \( \text{Ac}(t) \). But this is incompatible with \( \text{Ac}(\sim t) \) which we derive from (3a*) and (4a*) by Modus Ponens.

It is to be noted that no such paradox is known to have been advanced by anyone in confirmation theory. The reason is simply because the error upon which it is based is too glaring for anyone reasonably acquainted with inductive logic to commit. But then the error that generates the deontic paradox is basically the same except of course we cannot expect philosophers to be as sure-footed in deontic logic, which has only recently come into existence, as they are when dealing with the theory of probability, which is three hundred years old. This, however, is precisely why the central principle advocated in this paper is so vitally important: always make use of the well-established results of inductive logic as a reliable guide to show us the correct path to follow in deontic logic.

It is well known that 'the probability of \( (t \text{ if } g) \)' is correctly represented not by \( p(g \supset t) \) but by \( p(t/g) \). Consequently 'the probability of \( (t \text{ if } g) \) is equal to \( m' \) is to be denoted by '\( p(t/g) = m' \). But then of course \( p(t) = (g) \cdot p(t/g) + \ldots \ldots \). Therefore \( = m^2 + \ldots \ldots \) and no contradiction follows.

Similarly then Chisholm's (2) should have been symbolized not as \( (g \supset t) \) but rather by \( O(g/t) \). It is not that \( O(g \supset t) \) is an ill-formed expression. What it asserts, however, is that it is obligatory to see to it that either \( g \) is
false or $t$ is true. This implies that one can fully discharge one’s duty simply by not going or by merely telling that one is coming even though one is not. This crucial point of course has first been realized by Von Wright. However it so happens that in confirmation theory there would have been no room for hesitation to begin with since there we have conspicuous and entirely compelling reasons why ‘the probability of $t$ if $g$’ is not correctly represented by $\Pr(g \subseteq t)$ but by $\Pr(t/g)$. As is known the expression $g \subseteq t$ is true as soon as $g$ is false. It follows therefore that when $g$ is false, $\Pr(g \subseteq t) = 1$. But of course the value of the probability of $t$ given that $g$ is not necessarily 1 just because $g$ is false; the value of that probability depends essentially on the relationship that exists between $g$ and $t$. The probability of $t$ given that $g$ is determined by the confidence we may have in the truth of $t$ on being given the truth of $g$ or alternatively it is determined by the frequency of cases in which $t$ is true under circumstances in which $g$ is true. Thus we use the expression $\Pr(t/g)$ the value of which is not determined either by the truth value of $g$ alone or that of $t$ alone but on the degree to which $g$ probabilifies $t$.

Had our principle been adopted right from the beginning as the chief guide in deontic logic no paradox throwing a wrench into the smooth development of the formalization of moral discourse would have arisen in the first place. In that case Chisholm’s premise (2) would have been symbolized as $O(t/g)$. But then there would have been no way to derive $O(t)$. As has been demonstrated in Section VIII the relation ‘——— generates the obligation to ———’ is not transitive. Consequently $O(g) \& O(t/g) \rightarrow O(t)$ does not hold.

(X)

With the exception of Section II and the last paragraph of Section VI, the support offered throughout this paper for which I claim to be the central principle of deontic logic, amounts to no more than inductive evidence. That is, we have reviewed a considerable number of theorems in the logic of moral obligations and found all of them conforming to our principle, without exception. We have also seen how a variety of problems and paradoxes disappear when that principle is applied to them. A sympathetic reader may after looking at one or two further examples, become fully convinced that the principle is correct. However, we are not confronted here with a law of physics or chemistry which is true simply because our universe happens to be the way it actually is, but a statement which is true because it is conceptually required to be so. Thus we should be dissatisfied if we could support the principle merely by citing instance after instance that accords with it, without providing it with a theoretical defense.
Let me conclude this paper by indicating, albeit briefly, and in outline, the ultimate conceptual basis of our principle. If ‘O(A/B)’ stands for ‘I am obliged to bring about A, given that B’, then there is always an empirical statement of E — to the nature of which I shall come presently — such that, I am rationally justified in accepting it on the basis of B, if and only if, O(A/B) holds. In symbols:

(Φ): O(A/B) ≡ Ac(E/B)

The central principle of deontic logic is transparently implicit in expression (Φ). It follows immediately from (Φ) that all the truth conditions of O(A/B) are identical with those of Ac(E/B).

Now a Utilitarian, for example, would let ‘E’ stand for something like ‘My not bringing about A involves injury or loss to others of a significant magnitude while in comparison the cost of my doing A, is small’.

Since there are many complex moral theories, it would be pointless to attempt to rephrase E in the terms of each theory. But there is no need to. After all, hardly anyone wishes to claim that given two indiscernible state of affairs, the moral obligations arising in them may not be identical. (Φ) is true relative to any reasonable theory, where ‘E’ stands for a rather complex statement, including descriptions of many initial physical and psychological conditions and the laws of nature relevant to the predicted effects of A. For in the context of any given ethical theory there is always a relevant conjunction of empirical statements that uniquely determines the obtaining moral duties.

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11 On this view of course two factors are responsible for the variations in the degree to which an obligation may obtain: (i) the variations in the degree of acceptance of E (ii). The variations in the difference between the magnitude of the loss to others and the cost to me.